Target space duality and moduli stabilization in String Gas Cosmology

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Abstract

Motivated by string gas cosmology, we investigate the stability of moduli fields coming from compactifications of string gas on torus with background flux. It was previously claimed that moduli are stabilized only at a single fixed point in moduli space, a self-dual point of T-duality with vanishing flux. Here, we show that there exist other stable fixed points on moduli space with non-vanishing flux. We also discuss the more general target space dualities associated with these fixed points.

1 Introduction

String gas cosmology pioneered by Brandenberger and Vafa [1] is one of the attempts to apply string theory in cosmology. The advantage of this model is that it can provide a solution to the initial singularity problem and can explain the dimensionality of space-time. The universe in string gas cosmology starts from a very small, dense, and hot state where the matter content is dominated by gas of closed strings. All 9-dimensional spatial dimensions are taken to be toroidal compactified at the radius of string length.

The target space duality (T-duality) [2] in string theory implies the minimum length scale that we can probed. This give us a possible solution to avoid the initial singularity at the time of the big bang. The importance of the winding modes corresponding to closed strings wrapped around a circle of compactified dimension is also recognized. To get the model with expanding universe, the winding modes need to be annihilated. The dimensionality of the universe is the result of decompactification of 3 out of 9 spatial dimensions through winding modes annihilation process [3, 4]. The extension of this cosmological consideration to toroidal orbifolds was considered in [5]. These nice features make string gas very attractive. Its brane generalization is investigated in [6, 7, 8, 9, 10, 11, 12, 13, 14].

The purpose of this paper is to investigate the stability of the the moduli fields which describe the volume and the shape of the unobserved compact extra-dimensions and the background antisymmetric field. This is known as the moduli problem in string gas cosmology (for a review see [15]). Previous works on moduli stabilization of string gas compactification show that the moduli can be stabilized at the self-dual fixed-point with vanishing antisymmetric field. Numerical evidence for stability of the volume moduli was shown by Brandenberger and Watson but they find that the dilaton is running logarithmically [16]. The effective field theory approach shows that the dilaton and the radion can not be stabilized except for the 5-dimensional case [17].

The important roles of string massless modes in stabilization mechanism was discovered by Patil and Brandenberger [18] and the stability of the volume moduli can be proved analytically. The Higgs-like mechanism for stabilizing moduli at points of enhanced gauge symmetry was proposed in [19]. Recently, Kanno and Soda [20] considered the 4-dimentional effective action by taking into account T-duality. They have shown that the dilaton is marginally stable as the dilaton potential disappears when the moduli is stabilized at the self-dual radius. In this paper, we suggest there should exist other stable fixed-points on the moduli space of string gas compactification with non-zero anti-symmetric field background.

This paper is organised as follows: in section 2 we review the compactification of closed and heterotic string on a torus with backgroud antisymmetric field by following the works of Narain et al. [21, 22]. In section 3 we discuss target space duality symmetries of toroidal compactification and their fixed-points. We express the 4 dimensional T-duality invariant effective action in section 4. In section 5 by considering 4 dimensional effective action for a gas of heterotic string compactified on $T^2 \times T^2 \times T^2$, we show the stability of moduli at the fixed-points. We also discuss the stability of the dilaton field. In section 6 we present our conclusions.

2 Closed string on lattice torus

Consider a closed string propagating on a torus with metric G_{IJ} and a background anti-symmetric field B_{IJ} described by the action:

$$S = \frac{1}{4\pi\alpha'} \int d\tau d\sigma \left[\sqrt{-\gamma}\gamma^{ab} G_{IJ} \partial_a X^I \partial_b X^J + \epsilon^{ab} B_{IJ} \partial_a X^I \partial_b X^J \right], \tag{1}$$

where X^I $(I = 1 \cdots d)$ represent the toroidal coordinates and we ignore the non-compact directions for the moment. As we will see later, it is convenient to encode all the geometric data of the compact background in the "background matrix" E where

$$E_{IJ} = G_{IJ} + B_{IJ}. \tag{2}$$

Let us take the world-sheet metric $\gamma^{ab} = \eta^{ab}$ and X^I to be normalized such that X^I and $X^I + 2\pi n^I$ are the same point on the torus for any integer winding number n^I. The canonical momentum P_I associated with X^I is given by

$$2\pi\alpha' P_I = G_{IJ} \dot{X}^J + B_{IJ} X'^J = p_I + oscillators, \tag{3}$$

where the string center of mass momentum p_I is quantized in integer units $p_I = m_I$ due to the periodicities of X^I coordinates. m_I is the momentum number. The coordinates X^I can be split into left-moving part and right- moving part, $X^I = X^I_L + X^I_R$. By following [22] the mode expansions for X^I_L, and X^I_R can be written as

$$\begin{aligned} X^I_L &= \frac{x^I_L}{2} + \sqrt{\frac{\alpha'}{2}} \alpha^I_0 (\tau + \sigma) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \alpha^I_n e^{-in(\tau+\sigma)}, & (4) \\ X^I_R &= \frac{x^I_R}{2} + \sqrt{\frac{\alpha'}{2}} \tilde{\alpha}^I_0 (\tau - \sigma) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{n} \tilde{\alpha}^I_n e^{-in(\tau-\sigma)}. & (5) \end{aligned}$$

The background fields G_{IJ} and B_{IJ} only affect the quantization of string zero-modes which can be expressed in terms of left and right moving momenta, $\alpha^I_0 = \sqrt{\frac{\alpha'}{2}} p^I_L$ and $\tilde{\alpha_0}^I = \sqrt{\frac{\alpha'}{2}} p^I_R$, with

$$\begin{aligned} p^I_L &= G^{IJ} m_J + \frac{n^I}{\alpha'} - G^{IJ} B_{JK} \frac{n^K}{\alpha'}, \\ p^I_R &= G^{IJ} m_J - \frac{n^I}{\alpha'} - G^{IJ} B_{JK} \frac{n^K}{\alpha'}, \end{aligned} \tag{6}$$

The mode expansion of conjugate momenta reads

$$2\pi\alpha' P_I = p_I + \sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}[E_{IJ}\alpha_n^J e^{-in(\tau+\sigma)} + E_{IJ}^T\tilde{\alpha}_n^J e^{-in(\tau-\sigma)}]. \tag{7}$$

By expanding the equal-time commutation relation $[X^I(\sigma,0), P_J(\sigma',0)] = i\delta_J^I\delta(\sigma-\sigma')$, the non-zero commutation relations for α_n^I, $\tilde{\alpha}_n^I$, $x^I = \frac{1}{2}(x_L^I + x_R^I)$ and p_I are

$$[x^I, p_J] = i\delta_J^I \quad , \quad [\alpha_n^I, \alpha_m^J] = [\tilde{\alpha}_n^I, \tilde{\alpha}_m^J] = mG^{IJ}\delta_{m+n,0}. \tag{8}$$

Here the oscillators and their commutation relation are background dependent. The Virasoro generators are defined by

$$L_0 = \frac{1}{2}\alpha_0^I\alpha_0^I + N \quad , \quad \tilde{L}_0 = \frac{1}{2}\tilde{\alpha}_0^I\tilde{\alpha}_0^I + \tilde{N}, \tag{9}$$

where the number operators:

$$N = \sum_{n>0}\alpha_{-n}^I G_{IJ}\alpha_n^J \quad , \quad \tilde{N} = \sum_{n>0}\tilde{\alpha}_{-n}^I G_{IJ}\tilde{\alpha}_n^J. \tag{10}$$

we can define vectors e_I^i to be a basis to the compactification lattice Λ^d, such that the d-dimensional torus is $T^d = R^d/\pi\Lambda^d$. A basis to the dual lattice Λ^{d*} is denoted by e_i^{*I}. The indices i,j label an orthonormal basis to the torus. The scalar products of vectors e_I^i and e_i^{*I} are:

$$\sum_{i=1}^d e_I^i e_J^i = G_{IJ}, \;\; \sum_{i=1}^d e_i^{*i} e_i^{*J} = G^{IJ}, \;\; \sum_{i=1}^d e_I^i e_i^{*J} = \delta_I^J. \tag{11}$$

In this basis, the momenta in (6) are:

$$\begin{aligned} p_{Li} &= [m_I + E_{IJ}^T\frac{n^J}{\alpha'}]e_i^{*I}, \\ p_{Ri} &= [m_I - E_{IJ}\frac{n^J}{\alpha'}]e_i^{*I}, \end{aligned} \tag{12}$$

To ensure modular invariance of the closed string spectrum, it is required that $\alpha_0^2 - \tilde{\alpha}_0^2 = m^I n_I \in 2\mathbb{Z}$. This implies that the vectors $(\sqrt{\frac{\alpha'}{2}}p_{Li}, \sqrt{\frac{\alpha'}{2}}p_{Rj})$ span even self-dual (d,d) Lorentzian lattice $\Gamma^{(d,d)}$, with negative (positive) signature for left (right) momenta. The mass formula for closed string is given by:

$$m^2 = \frac{1}{2}(p_L^2 + p_R^2) + \frac{2}{\alpha'}(N + \tilde{N} - 2), \tag{13}$$

with the level matching condition $L_0 - \tilde{L}_0 = 0$ which can be written as:

$$N - \tilde{N} = m_I n^I. \tag{14}$$

For generic toroidal compactifications, the massless vectors $\alpha_{-1}^\mu\tilde{\alpha}_{-1}^I|0_L, 0_R\rangle$ and $\tilde{\alpha}_{-1}^\mu\alpha_{-1}^I|0_L, 0_R\rangle$, where the μ and I indices respectively label the non-compact and compact dimensions, generate a local $[U(1)]^d \times [U(1)]^d$ gauge symmetry. As it was first pointed out in [21], more massless vectors arise when $\sqrt{\frac{\alpha'}{2}}p_{Li}$ and $\sqrt{\frac{\alpha'}{2}}p_{Ri}$ are roots of the simply laced group $\mathcal{G}_L$ and $\mathcal{G}_R$ of rank d with root length $\sqrt{2}$. The gauge symmetry is enhanced to $\mathcal{G}_L \times \mathcal{G}_R$.

Note that in the case of heterotic string, we can include 16 left-moving bosons propagating in maximal torus of $E_8 \times E_8$ or $Spin(32)/\mathbb{Z}_2$. The resulting $\Gamma^{(16+d,d)}$ lattice is still an even self-dual Lorentizian lattice. However, this extra degree of freedoms will not affect our later considerations. As you will see later in section 5, it is enough for us to concentrate on the $d \times d$ dimensional sublattice of the full $\Gamma^{(16+d,d)}$ lattice. The mass formula for heterotic string is:

$$m^2 = \frac{1}{2}(p_L^2 + p_R^2) + \frac{2}{\alpha'}(N + \tilde{N} - 1), \tag{15}$$

with matching condition

$$N - \tilde{N} + 1 = m_I n^I. \tag{16}$$

Note that in this case N and $\tilde{N}$ are the number operators for (26-dimensional) left-handed oscillators and (10-dimensional) right-handed oscillators respectively. We can also consider enhancement of gauge symmetry from heterotic string compactified on d-dimensional torus. For generic points in moduli space, the gauge symmetry is $U(1)_L^{16+d} \times U(1)_R^d$. However some of the $U(1)$ can be enhanced to non-abelian gauge groups. This will happen when the lattice $\Gamma^{(16+d,d)}$ contains vectors with $(\sqrt{\frac{\alpha'}{2}} p_L)^2 = 2$ and $\sqrt{\frac{\alpha'}{2}} p_R = 0$. The vectors $\sqrt{\frac{\alpha'}{2}} p_{Li}$ are roots of the simply laced group $\mathcal{G}_L$ of rank $16 + d$ with root length $\sqrt{2}$. The largest enhanced symmetry gauge group that we can have is $\mathcal{G}_L = SO(32 + 2d)$.

3 Target space duality and their fixed points

We are now at the position to discuss the target space duality in closed string theory and examine their fixed-points. The target space duality will work in a similar way as in the heterotic string case. Let us start by rewriting closed string mass formula (13) in a $O(d, d, \mathbb{Z})$ covariant form (in $\alpha' = 1$ unit) as:

$$m^2 = Z^T M(E) Z + 2(N + \tilde{N} - 2). \tag{17}$$

The matching condition is also rewritten as $N - \tilde{N} = \frac{1}{2} Z^T \eta Z$, where we define

$$M(E) = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix} \;,\; \eta = \begin{pmatrix} 0 & \mathbf{1}_{d\times d} \\ \mathbf{1}_{d\times d} & 0 \end{pmatrix}, \tag{18}$$

and $Z = (m_1, \ldots, m_d, n_1, \ldots, n_d)$. It is obvious that the closed string spectrum is invariant under $O(d, d, \mathbb{Z})$ transformation:

$$M \to \Omega M \Omega^T \;,\; Z \to \Omega Z, \tag{19}$$

where $\Omega \in O(d, d, \mathbb{Z})$ is the integer valued matrix satisfying $\Omega^T \eta \Omega = \eta$.

We would like to discuss a particular element $\Omega = \eta$ of $O(d, d, \mathbb{Z})$. This is known as the T-duality. It is given by the inversion of the background matrix E [25, 26]:

$$E \to E' = G' + B' = E^{-1}. \tag{20}$$

Here we take G' and B' to be the symmetric and antisymmetric part of E' respectively. Note that the duality maintains the symmetry (anti-symmetry) of G (B). Under T-duality the metric and the antisymmetric field transform as

$$\begin{aligned} G \to G' = (G - BG^{-1}B)^{-1}, B \to B' = (B - GB^{-1}G)^{-1}, \\ G^{-1}B \to -BG^{-1}. \end{aligned} \tag{21}$$

The closed string mass spectrum (13) is manifestly invariant under transformation (21) together with the interchange of winding modes with momentum modes.

Let us consider the fixed point of T-duality by starting with the special case where the antisymmetric B field is absence. We obtain the transformation:

$$G \to G^{-1}. \tag{22}$$

For one compact dimension ($d = 1$), one can think of a closed string moving in the circle of radius R. T-duality $R \to \frac{1}{R}$ has a fixed point at the self-dual radius $R = 1$. At this fixed point the gauge symmetry is enlarged form $U(1)_L \times U(1)_R$ to $SU(2)_L \times SU(2)_R$.

For more general compactification, the $E \to E^{-1}$ duality still has a single fixed point at

$$G = \mathbf{1}_{d\times d}, \quad B = 0, \tag{23}$$

which is the unique solution to the equation $(G+B)^2 = \mathbf{1}_{d\times d}$ for positive definite G. We can show that the gauge symmetry get enhanced to $SU(2)^d_L \times SU(2)^d_R$.

However, there exists a more general example of fixed-points of T-duality modulo $SL(d,\mathbb{Z})$ and $\Theta(\mathbb{Z})$ transformation [23, 25, 26]:

$$E^{-1} = M^T(E+\Theta)M, \; , \; M \in SL(d,\mathbb{Z}), \; \Theta \in \Theta(\mathbb{Z}), \tag{24}$$

where $\Theta(\mathbb{Z})$ is the symmetry transformation that adds to B_{IJ} an antisymmetric integral matrix Θ_{IJ} ($\Theta_{IJ} \in \mathbb{Z}$ and $\Theta_{IJ} = -\Theta_{JI}$). A class of background with a maximally enhanced symmetry gives an example of such fixed points [24]. This background are characterized by $e^i_I = \sqrt{\frac{\alpha'}{2}} r^i_I$ where the vector r^i_I span the root lattice Λ_{root} of $\mathcal{G}$ and are chosen to be the simple roots with $(r_I)^2 = 2$. It follows from (11) that $\frac{2}{\alpha'}G_{IJ}$ is the Cartan matrix of $\mathcal{G}$. To be more precise, we choose the metric and the antisymmetric field as:

$$G_{IJ} = \frac{1}{2}C_{IJ}, \;\; B_{IJ} = \begin{cases} G_{IJ} & , I<J \\ 0 & , I=J \\ -G_{IJ} & , I>J \end{cases}, \tag{25}$$

where C_{IJ} is the Cartan matrix of $\mathcal{G}$. Since $E, E^{-1} \in SL(d,\mathbb{Z})$, the solution to (24) is obtained by taking $M = E^{-1}$ and $\Theta = E^T - E$.

Let us consider the $d=2$ example. There are two fixed points of enhanced symmetry. The first is the fixed point of $SU(2)^2_L \times SU(2)^2_R$ symmetry as in (23). The second is the $SU(3)_L \times SU(3)_R$ fixed point which the background matrix E, G and antisymmetric field B are:

$$E_{IJ} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \;\; G_{IJ} = \frac{1}{2}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}, \;\; B_{IJ} = \frac{1}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{26}$$

or equivalently, we can choose the basis vector in (11) to be:

$$e^i_1 = \frac{1}{2}(1,\sqrt{3}), \;\; e^i_2 = \frac{1}{2}(1,-\sqrt{3}). \tag{27}$$

Under duality (20), we have

$$E^{-1}_{IJ} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \;\; G'_{IJ} = \frac{1}{2}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \;\; B'_{IJ} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{28}$$

It is obvious that $E^{-1} \neq E$ at this fixed point. The importance of this fixed point can be observed by considering the effect of T-duality on the $SU(3)$ basis vectors. This effect is called the E-duality transformation in [28] and the basis vectors are transformed as:

$$e^i_I \to E^I_i, \;\; e^{*I}_i \to E^{*i}_I. \tag{29}$$

Here we can choose:

$$E^1_i = \frac{1}{2}(1,\sqrt{3}), \;\; E^2_i = (1,0). \tag{30}$$

One can see that the basis (27) and the E-dual basis (30) span the same $SU(3)$ lattice (see Figure1). Hence, T-duality transformation changes only the basis vectors of the lattice without changing the lattice itself.

In the case of heterotic string compactified on $d=2$ dimensioanl torus, the background in (23) is associated with the self-dual fixed point with enhanced gauge symmetry $SU(2)\times SU(2)$. The metric and antisymmetric field in (26) correspond to the fixed-point with $SU(3)$ enhanced gauge symmetry. We will call them them the $SU(2)\times SU(2)$ and the $SU(3)$ fixed-point respectively.

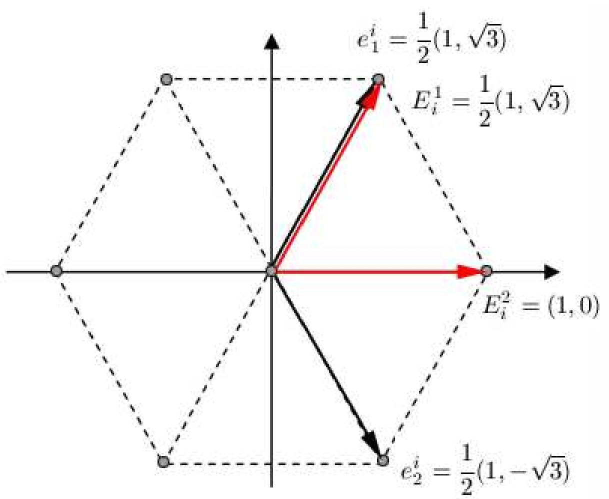

Figure 1: The basis e^i_I and the E-dual basis E^I_i span the same $SU(3)$ lattice.

4 Low energy effective action for string gas

The bosonic part of the low energy effective action for closed string is given by

$$S = -\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-G_{10}}\, e^{-2\phi}(R^{(D)} + 4\partial^A\phi\partial_A\phi - \frac{1}{12}H^2), \tag{31}$$

where $H = dB$ is the field strength for the antisymmetric field, κ is the 10 dimensional gravitational coupling constant. G_{10} and ϕ represent the 10-dimensional metric and the dilaton respectively. The indices $A = 0, 1, \ldots, 9$.

There are some assumptions we would like to make. First, we apply the Brandenberger-Vafa mechanism such that the four dimensional space-time is practically non-compact while the 6-dimensional internal space is toroidally compactified. Second, we assume the cosmological ansatz :

$$ds^2 = g_{\mu\nu}(x^\mu)dx^\mu dx^\nu + G_{IJ}(x^\mu)dy^I dy^J. \tag{32}$$

Here, the metric G_{IJ} represent the d-dimensional compact space and $g_{\mu\nu}$ is the metric for the 4 dimensional non-compact space-time with $\mu, \nu = 0, \ldots, 4$. We also assume the metric depends only on the non-compact coordinates x^μ. For simplicity, we choose the anti-symmetric field to have non-vanishing components only in the compact directions. The action (31) can be written as:

$$S = -\frac{V_6}{2\kappa^2}\int d^4x e^{-\Phi}(R^{(4)} + \partial_\mu\Phi\partial^\mu\Phi + \frac{1}{8}Tr[\partial_\mu M\eta\partial^\mu M\eta]), \tag{33}$$

where the shift dilaton filed is defined by:

$$\Phi = 2\phi - \ln\sqrt{G}. \tag{34}$$

It is easy to show that the action (33) is invariant under T-duality [27].

The action for a gas of string is given by

$$S_{gas} = \mu_4\int d^4x\sqrt{-g_{00}}\,\rho, \tag{35}$$

where μ_4 is the comoving number density of a string gas and the energy ρ is defined by

$$\rho = \sqrt{g^{\mu\nu}p_\mu p_\nu + m^2(E)}. \tag{36}$$

It is easy to see that the string gas action is also T-duality invariant. In order to consider the stability, we choose to work in the Einstein frame by defining the Einstein metric $(g_E)_{AB} = e^{-\Phi} g_{AB}$. The full action in the Einstein frame is

$$\begin{aligned} S_E = & - \frac{V_6}{2\kappa^2} \int d^4x \sqrt{-g_E}\, (R_E^{(4)} + g_E^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} Tr[\partial_\mu M \eta \partial^\mu M \eta]) \\ & + \int d^4x \sqrt{-g_E}\, V_{eff}(g_E, \Phi, E), \end{aligned} \tag{37}$$

where the effective potential is,

$$V_{eff}(g_E, \Phi, E) \equiv \frac{\mu_4}{\sqrt{g_E^s}} \sqrt{g_E^{\mu\nu} p_\mu p_\nu + e^\Phi m^2(E)}. \tag{38}$$

5 Stability of heterotic string gas compactified on torus

In order to avoid instability from the presence of tachyonic states [29], we study a model of supersymmetric string gas. Since toroidal compactification of Type II superstring does not give us enhanced gauge symmetry, we will consider the SO(32) heterotic string gas compactified on $T^2 \times T^2 \times T^2$.

Concentrating on the compact part, we can analyze each torus separately. As we have seen in the $d = 2$ example, there must exist two fixed points with enhanced symmetry in the moduli space of each 2-dimensional torus. The first is the $SU(2) \times SU(2)$ fixed-point and the second is the $SU(3)$ fixed point. We would like to show that these two fixed points are stable.

We will follow the method used in [20] to analyze the stability of the internal compacted space. Since internal space is the direct product of the torus, we could simplify the problem a little by considering only one of the three tori. More precisely, we consider a 6 dimensional space with 2 dimension toroidal compactified. Let us use the metric ansatz in (32):

$$ds^2 = g_{\mu\nu}(x^\mu) dx^\mu dx^\nu + ds^2_{torus}. \tag{39}$$

The line element for the torus part is written as

$$ds^2_{torus} = \frac{b^2}{\eta}[(dy^1 + \xi dy^2)^2 + \eta^2 (dy^2)^2], \tag{40}$$

with the background antisymmetric field,

$$B = \begin{pmatrix} 0 & \beta \\ -\beta & 0 \end{pmatrix}. \tag{41}$$

Here we can see that the moduli space is described by 4 parameters, b^2, η, ξ, and β. The parameter b plays the role of the scale factor of the torus. While parameters η and ξ control the shape of the moduli. The last parameter β is called the flux moduli, it determines the value of the antisymmetric background.

We are interested in heterotic string states whose mass vanishes at the fixed point. Recall the mass formula for heterotic string in (15)in $\alpha' = 1$ unit:

$$m^2 = \frac{1}{2}(p_L^2 + p_R^2) + 2(N + \tilde{N} - 1), \tag{42}$$

with matching condition $N - \tilde{N} + 1 = m_A n^A \in \mathbb{Z}$. p_{Li} and p_{Ri} are defined in (12). For simplicity, we turn off every oscillators and zero-modes in SO(32) directions. This can be done without affecting the final results. Since we consider a particular torus T^2 subspace of $T^2 \times T^2 \times T^2$, we must keep only the vector p_{Li} that is embedded in that subspace. At the enhanced symmetry point of toroidal compactification, the zero-modes $\sqrt{\frac{1}{2}} p_{Li}$, are roots of the simply laced group G_L of rank 2 with root length $\sqrt{2}$. The gauge symmetry gets enhanced to G_L.

5.1 $SU(2) \times SU(2)$ fixed point

By comparing (40) and (41) with (23), it is easy to see that the self-dual fixed point with enhanced gauge symmetry $G_L = SU(2) \times SU(2)$ has

$$b^2 = \eta = 1, \beta = \xi = 1. \tag{43}$$

We will label the massless string modes by their momenta and winding numbers in y^1 and y^2 directions, (m_1, m_2, n^1, n^2). At the $SU(2) \times SU(2)$ fixed-point, the four heterotic string states that become massless are:

$(m_1,$	$m_2,$	$n^1,$	$n^2)$
± 1	0	± 1	0
0	± 1	0	± 1

At very near to the $SU(2) \times SU(2)$ fixed point, these four states are not exactly massless and each mode contributes to the effective potential (38). In order to prove that the $SU(2) \times SU(2)$ fixed-point is a stable minimum of the effective potential, we can consider the minimum of m^2 in (38). Let us examine the heterotic string gas mode by mode.

Mode (1, 0, 1, 0)
This mode corresponds to closed strings wrapped around a circle in the y^1 direction. It has the mass squared

$$m^2_{1010} = \frac{1}{b^2\eta}(1 + \beta\xi)^2 + \frac{\beta^2\eta}{b^2} + \frac{b^2\xi^2}{\eta} + \eta b^2 - 2, \tag{44}$$

with flat directions $\beta^2 = \frac{b^4\xi^2}{\eta^2}$ and $1 + \beta\xi = \eta b^2$.

Mode (0, 1, 0, 1)
The mass squared for this mode which corresponds to closed strings wrapped around a circle in the y^2 direction is given by

$$m^2_{0101} = \frac{1}{b^2\eta}(\xi - \beta)^2 + \frac{b^2}{\eta} + \frac{\eta}{b^2} - 2. \tag{45}$$

There exists the flat directions $\beta = \xi$ and $b^2 = \eta$.

We find these two flat directions intersect at the self-dual $SU(2) \times SU(2)$ fixed point $b = \eta = 1$, $\xi = \beta = 0$. Hence, by taking into account both modes of string gas, the self-dual point would be a stable minimum. The stability can be explicitly verified by expanding the potential around this fixed-point. This was already discussed by the authors of [20]. They showed that, by considering contributions from both massless modes, the flat directions of the effective potential will disappear if we move away from the $SU(2) \times SU(2)$ fixed-point. This implies that the volume, shape, and flux moduli get stabilized at the self dual point. The result will not alter if we add the contributions from $(-1, 0, -1, 0)$ and $(0, -1, 0, -1)$ modes. Let us investigate another fixed point.

5.2 $SU(3)$ fixed point

Let us consider the fixed point with enhanced symmetry $G_L = SU(3)$. By comparing (40) and (41) with (26), this fixed-point has

$$b^2 = \eta = \frac{\sqrt{3}}{2}, \beta = \xi = -\frac{1}{2}. \tag{46}$$

At this point on moduli, six heterotic string modes become massless:

$(m_1,$	$m_2,$	$n^1,$	$n^2)$
0	± 1	0	± 1
± 1	0	± 1	± 1
± 1	∓ 1	± 1	0

As we approach the $SU(3)$ fixed-point, these six modes are turned on. Their mass will contribute to the effective potential. Using the similar investigation to the self-dual fixed-point, it is enough for us to consider just three massless modes:

Mode (0, 1, 0, 1)
The modes correspond to closed strings wrapped around a circle in the y^2 direction are massless at both the $SU(2)\times SU(2)$ and $SU(3)$ fixed-points. Their contributions to the effective potential is

$$m^2_{0101} = \frac{1}{b^2\eta}(\xi-\beta)^2 + \frac{b^2}{\eta} + \frac{\eta}{b^2} - 2, \tag{47}$$

with flat directions $\beta=\xi$ and $b^2=\eta$.

Mode (1, 0, 1, 1)
This mode represents string wrapped on both circles of the torus. Its mass squared can be written as:

$$m^2_{1011} = \frac{1}{b^2\eta}(1+\beta\xi)^2 + \frac{\eta\beta^2}{b^2} + \frac{b^2}{\eta}(\xi+1)^2 + \eta b^2 + \frac{\beta^2}{b^2\eta}(\beta+2\beta\xi+2) - 2. \tag{48}$$

It has flat directions $(\xi+1) = -\frac{\beta}{\beta^2+b^4}$ and $(\xi+1)^2 = \frac{\eta^2\beta^2}{b^4}$.

Mode (1, -1, 1, 0)
This mode denotes string wrapped on a circle in y^1. Its mass squared is:

$$m^2_{1-110} = \frac{1}{b^2\eta}(1+\beta\xi)^2 + \frac{\eta}{b^2}(1+\beta)^2 + \frac{b^2}{\eta}(\eta^2+\xi^2) + \frac{\xi^2}{b^2\eta}(\xi+2\beta\xi+2) - 2. \tag{49}$$

The mass squared has flat directions $(\beta+1) = -\frac{\xi}{\xi^2+\eta^4}$ and $(\beta+1)^2 = \frac{b^4\xi^2}{\eta^4}$.

These three flat directions intersect at the $SU(3)$ fixed point: $b^2=\eta=\frac{\sqrt{3}}{2}$ and $\beta=\xi=-\frac{1}{2}$. The stability can be explicitly verified by perturbing the metric around the minimum of the effective potential. For small m^2, we get

$$V = \mu_4\sqrt{\frac{g^{ij}p_ip_j}{g_s}} + \frac{1}{2}\frac{\mu_4 e^{2\phi}}{\sqrt{g^{ij}p_ip_j}}m^2(\beta,b,\eta,\xi). \tag{50}$$

For convenience, we change coordinates to:

$$b^2 = \frac{\sqrt{3}}{2}\bar{b}^2,\ \xi = -\frac{1}{2}\bar{\xi},\ \eta = \frac{\sqrt{3}}{2}\bar{\eta},\ \beta = -\frac{1}{2}\bar{\beta}. \tag{51}$$

In the new coordinates, the $SU(3)$ fixed-point is at

$$\bar{b} = \bar{\eta} = \bar{\beta} = \bar{\xi} = 1. \tag{52}$$

Let us expand the moduli around the $SU(3)$ fixed-point:

$$\bar{b} = 1+\delta\bar{b},\ \bar{\xi} = 1+\delta\bar{\xi},\ \bar{\eta} = 1+\delta\bar{\eta},\ \bar{\beta} = 1+\delta\bar{\beta}. \tag{53}$$

We get

$$\delta m^2_{0101} = \frac{1}{3}(\delta\bar{\xi}-\delta\bar{\beta})^2 + (\delta\bar{\eta}-2\delta\bar{b})^2, \tag{54}$$

$$\delta m^2_{1011} = \frac{1}{3}(\delta\bar{\xi}^2+\delta\bar{\xi}\delta\bar{\beta}+\delta\bar{\beta}^2) + (\delta\bar{\eta}^2+2\delta\bar{\eta}\delta\bar{b}+4\delta\bar{b}^2) + \delta\bar{\eta}\delta\bar{\beta} - 2\delta\bar{\xi}\delta\bar{b}, \tag{55}$$

$$\delta m^2_{1-110} = \frac{1}{3}(\delta\bar{\xi}^2+\delta\bar{\xi}\delta\bar{\beta}+\delta\bar{\beta}^2) + (\delta\bar{\eta}^2+2\delta\bar{\eta}\delta\bar{b}+4\delta\bar{b}^2) - \delta\bar{\eta}\delta\bar{\beta} + 2\delta\bar{\xi}\delta\bar{b}. \tag{56}$$

We can easily observe that each potential has flat directions. However, by summing all modes together, flat directions disappear in the total potential:

$$\delta m^2_{su(3)} = \delta m^2_{0101} + \delta m^2_{1011} + \delta m^2_{1-110} = \delta\bar{\beta}^2 + \delta\bar{\xi}^2 + 3\delta\bar{\eta}^2 + 3\delta\bar{b}^2. \tag{57}$$

This implies that the $SU(3)$ fixed point is a stable fixed point as we expected.

Let us summarize our results. The moduli can be stabilized at both fixed-points depending on the initial conditions. At the $SU(2)\times SU(2)$ fixed-point four modes, $(\pm1,0,\pm1,0)$ and $(0,\pm1,0,\pm1)$, are massless. If we move away from the fixed point, these modes become massive and contribute to the contracting potential. On the other hands, if the initial condition is near to the $SU(3)$ fixed point, we should turn off the $(0,\pm1,0,\pm1)$ modes as they are massive at the $SU(3)$ fixed point and will not contribute to the stabilization mechanism. The six massive modes $(\pm1,0,\pm1,0)$, $(\pm1,0,\pm1,\pm1,)$ and $(\pm1,\mp1,\pm1,0)$ will be turned on and play their roles in a contracting potential to stabilize the moduli at the $SU(3)$ fixed point.

We also have to consider the stability of the dilaton field. This can be done by considering the equations of motion from variations of the string frame action in (31). For simplicity, we will fix the shape of the torus and give constraints to the flux moduli as:

$$\frac{b^2}{\eta} = e^{2\upsilon},\ \beta = \xi\frac{b^2}{\eta} = \frac{z}{2}\,e^{2\upsilon},\ \xi^2+\eta^2 = 1. \tag{58}$$

Here υ is the function of the comoving time $t = x^0$ and z is a constant parameter with $z = 0$ and $z = -1$ representing $SU(2)\times SU(2)$ and $SU(3)$ fixed-points respectively. The 6 dimensional metric in (39) can be written as:

$$ds^2 = -dt^2 + e^{2\lambda(t)}\sum_{i=1}^{4} dx^i dx^i + e^{2\upsilon(t)}[(dy^1)^2 + z dy^1 dy^2 + (dy^2)^2], \tag{59}$$

with the background antisymmetric field,

$$B(t) = \frac{e^{2\upsilon(t)}}{2}\begin{pmatrix} 0 & z \\ -z & 0 \end{pmatrix}. \tag{60}$$

The equations of motion are:

$$\begin{aligned}
\ddot{\lambda} - 2\dot{\lambda}\dot{\phi} + 3\dot{\lambda}^2 + 2\dot{\lambda}\dot{\upsilon} &= \kappa^2 P_\lambda e^{2\phi} && (61)\\
\ddot{\upsilon} - 2\dot{\upsilon}\dot{\phi} + 3\dot{\lambda}\dot{\upsilon} + \frac{8}{4-z^2}\,\dot{\upsilon}^2 &= \kappa^2 P_\upsilon e^{2\phi} && (62)\\
2\ddot{\phi} - 4\dot{\phi}^2 + 6\dot{\phi}\dot{\lambda} + 4\dot{\phi}\dot{\upsilon} + \frac{z^2}{4-z^2}\dot{\upsilon}^2 &= \frac{\kappa^2}{2}T\,e^{2\phi}, && (63)
\end{aligned}$$

where $T = 3P_\lambda + 2P_\upsilon - \rho$ is the trace of the energy momentum tensor of string gas, $P_\lambda(P_\upsilon)$ denotes pressure in non-compact (compact) directions ,and a dot denotes a derivative with respect to time. By writing equation (63) as:

$$\frac{d^2}{dt^2}(e^{-2\phi}) + 3\dot{\lambda}\frac{d}{dt}(e^{-2\phi}) + 2\dot{\upsilon}\frac{d}{dt}(e^{-2\phi}) + \frac{1}{2}\frac{z^2}{4-z^2}\dot{\upsilon}^2(e^{-2\phi}) = -\kappa^2 T, \tag{64}$$

We come to the same conclusion as [20]. At the fixed-point $\upsilon = 0$ and $z = 0, -1$, the energy-momentum tensor of the string gas is traceless $T = 0$. The dilaton is stabilized by the hubble damping term due to the expansion of the non-compact direction. Since the dilaton can move along the flat direction, it is marginally stable at both fixed points.

6 Conclusions

We have shown that, for gas of string compactified on torus with an antisymmetric background field, there exist other fixed-points apart from the self-dual fixed-point. We have examined the

toy model of heterotic string gas compactified on the 6 dimensional internal space. We assume the compact space to be the direct product of three 2 dimensional tori. The stability of each torus can be investigated separately.

In the moduli of 2-dimensional torus, there are two fixed points. The first is the self-dual fixed point of T-duality. At this fixed-point, the heterotic string spectrum has $SU(2) \times SU(2)$ gauge symmetry. The second fixed point produces $SU(3)$ enhanced gauge symmetry to the string spectrum and it is a fixed point for the more general target space duality. More precisely, it is the fixed point for T-duality modulo the discrete rotational group and a shift on the B field with a constant integer value. We have shown that these two points are stable fixed-points on the moduli. The dilaton is marginally stable at both fixed-points. The existence of these fixed-points might have some application to cosmology. We leave this for future investigations.

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